VISUALIZING EQUIVALENT FRACTIONS

INTRODUCTION TO VISUALIZING RATIOS USING AN ONLINE SPIROGRAPH GENERATOR

Lesson Overview

This lesson is meant to be implemented alongside mathematics instruction covering equivalent fractions. Students are introduced to the **Inspiral Web Interface** https://nathanfriend.io/inspiral-web/ and use the website's simulation to learn how equivalent fractions work in a visual demonstration. Students also learn about gears, gear teeth and their simple machine function. This lesson was designed for 4th graders but can be used in older classes as well.

Project Description

Based on a problem-based learning design using the 5E Model of Instruction, students are challenged to create mathematics-based art that incorporates equivalent fractions and computer generated images. In addition to the basic construction of the images, students also predict the outcome of images based on their fraction sets using a basic knowledge of gears. This project is an introduction to using mathematics and engineering to create artistic imagery.

STEAM Integration

Engineering Simple Machines & Gears

Mathematics Using Equivalent Fractions to change image content

Visual Arts Use visual cues to spot similarities in: Repetition, Rhythm, and Pattern

Project Length

The project is expected to last 2-4 class periods. Lesson introduction and image construction take 1-2 class periods, and finalizing the images (Saving, Printing and Presenting) takes 1-2 class periods.

Learning Goals

STEM Learning Goals	Arts Learning Goals
- To use knowledge of equivalent fractions to generate images - To predicts similar imagery based on equivalent fraction sets - To produce images that reflect equivalent fraction sets - Understanding the basics of how gears work based on the number of teeth they have.	- To explore the interrelated principles between Art and Mathematics - To create and share similar Images using digital tools - To use visual cues to spot similarities in: Repetition, Rhythm, and Pattern
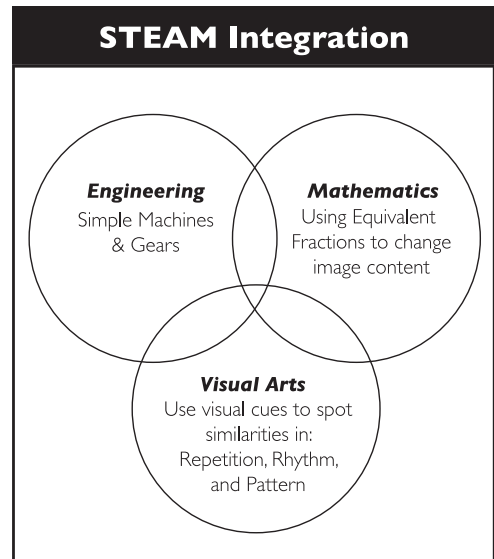
Lesson Vocabulary

Equivalent Fractions, Pattern, Rhythm, Repetition, Spirograph, Gears, Teeth, Ratio, Scale

Learning Standards

Mathematics
4.NR.4: Solve real-life problems involving addition, subtraction, equivalence, and comparison of fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100 using part-whole strategies and visual models. ● 4.NR.4.1 Using concrete materials, drawings, and number lines, demonstrate and explain the relationship between equivalent fractions, including fractions greater than one, and explain the identity property of multiplication as it relates to equivalent fractions. Generate equivalent fractions using these relationships. ● 4.NR.4.2 Compare two fractions with the same numerator or the same denominator by reasoning about their size and recognize that comparisons are valid only when the two fractions refer to the same whole. ● 4.NR.4.3 Compare two fractions with different numerators and/or different denominators by flexibly using a variety of tools and strategies and recognize that comparisons are valid only when the two fractions refer to the same whole.

Art

● VA4.CR.3: Understand and apply media, techniques, processes, and concepts of two-dimensional art.
  ○ a. Apply drawing and painting techniques with a variety of media (e.g. pencil, crayon, pastel, charcoal, tempera, watercolor, acrylic).
  ○ c. Combine materials in creative ways to make works of art (e.g. mixed-media, collage, available technology).
 	
● VA4.CR.5 Demonstrate an understanding of the safe and appropriate use of materials, tools, and equipment for a variety of artistic processes.

Background knowledge required of teachers

Teachers implementing the lesson should understand:
● Technical: The basic use of a spirograph, and the **Inspiral Web Interface** https://nathanfriend.io/inspiral-web/, and how to save and print images from the website.
● Mathematics: The manipulation of basic Equivalent Fraction sets
● Engineering/Science: The fundamentals of gears as a simple machine
● Visual Arts: Key Terms--Scale, Pattern, Rhythm, and Repetition

Materials

- Website: https://nathanfriend.io/inspiral-web/ (**Inspiral Web Interface)**
- Visualizing Equivalent Fractions Worksheet
- Glue, Tape, Scissors, Colorful Mounting Board (card stock or foam board)

ENGAGE: Making Connections

Section Goal: Students are introduced to the project's challenge and explore the concept of math-based art.
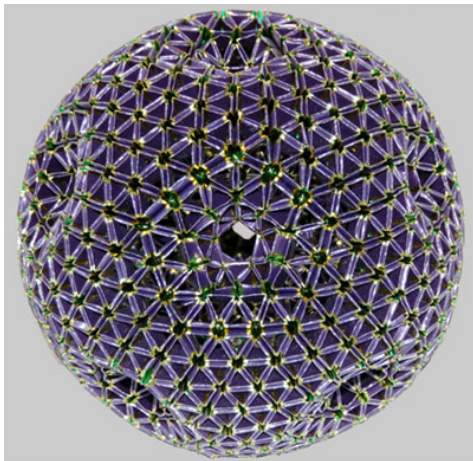
Section Activity:

Introductory Discussion: Begin the discussion by looking at math-based artwork that utilizes math that students are familiar with. The American Mathematics Society has many images on the website link provided. It is useful to project pictures of different types of math-based images. Emphasize the patterns that are part of math-based art.
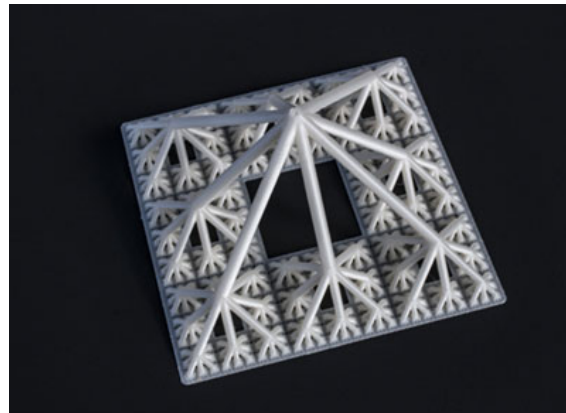
Possible questions for students:

Have you ever seen artwork that uses math before?
Do you see any patterns in the artwork?
How are they similar or different?
Why would an artist use math to make art?

Examples of simple math-based art.

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"Complex Carpet Octree," by Bernat Espigulé
(Holomorphic Dynamics Group, Universitat de Barcelona)

"Dodecahedral 11-Hole Torus," by Faye E. Goldman (Ardmore, PA)

Use of triangles, hexagons

Each point branches into 8 more. Repeated pattern.

"Makeri Mosaic," by Susan Goldstine
(St. Mary's College of Maryland, St. Mary's City)

"Diatoms I," by Phil Webster (Phil Webster Design, Scotts Valley, CA)

"The Right Rose," by Yana Mohanty (Imathgination LLC, San Diego, CA)
and Bjoern Muetzel (Dartmouth College, Hanover, NH)

Symmetry, repeated pattern, geometric patterns	Fractal, repeating geometric pattern	Arcs of circles, symmetry,

Introduce Student Challenge:

Your principal would like to exhibit some new art in the hallways and has asked your class to provide some math-based art that utilizes fractions and a computer program. So, you will be creating digital, math-based art.

In the next section, students will be introduced to gears and explore the spirograph website.

EXPLORE #1 – How gears work/Website Introduction

Section Goal: To understand the basics of how gears work and introduce students to the **Inspiral Web Interface** Spirography Generator (https://nathanfriend.io/inspiral-web/).

Section Activities:

1. **Provide an introduction to gears and how they work.**

   - Project on your screen the animation of two spinning gears.
     - https://commons.wikimedia.org/wiki/User:Jahobr/Gearboxes#/media/File:Animated_two_spur_gears_1_2.gif
   - Explain that a gear is a wheel and axle with teeth along the outside of the wheel. When combined with other gears, they make a **simple machine** that spins and can change the direction of a force and change how fast a second gear and axle rotates. The gears work together by using their teeth to move other gears.
   - How fast one gear can make another spin depends upon the difference in their number of teeth.

- In the example, the Green gear has twice as many teeth as the Red gear. This is called a Gear Ratio of 1 to 2 or 1:2.
- For every full **ONE** rotation of the **RED** gear, the **GREEN** gear has only rotated **half** of its full rotation.
- This means the **RED** gear must rotate **2 FULL** times before the **GREEN** gear has a full single rotation.
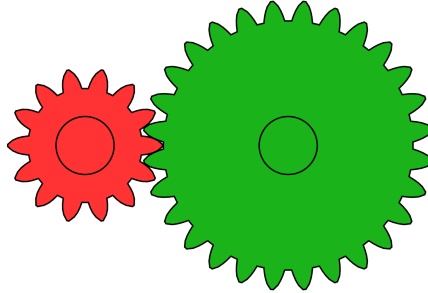

FIGURE 1: GEAR RATION 1:2 ANIMATION BY JAHOBR

- Gears are used in many different machines including things like bicycles, cars, boats, and watches. Here is a video that further explains how gears work:
  - https://www.youtube.com/watch?v=vX1-9C58-VM

2. **Introduce the Inspiral Web Interface** at https://nathanfriend.io/inspiral-web/ and explain the functions of the interface (as shown in the tutorials below):
  - How to choose the gear sets
  - How to choose colors (background and line color)
  - How to draw the image (drag mouse in circular motion)
  - How to save and print image

Using the Spirograph Interface:

1. First, choose a gear from the left-hand side. For this exercise, we will choose a LARGER gear for the **Fixed gear** and a SMALLER gear for the **Rotating gear**.
2. Next, choose your line color and your background color. This example will use blue, but you can choose a color you like.
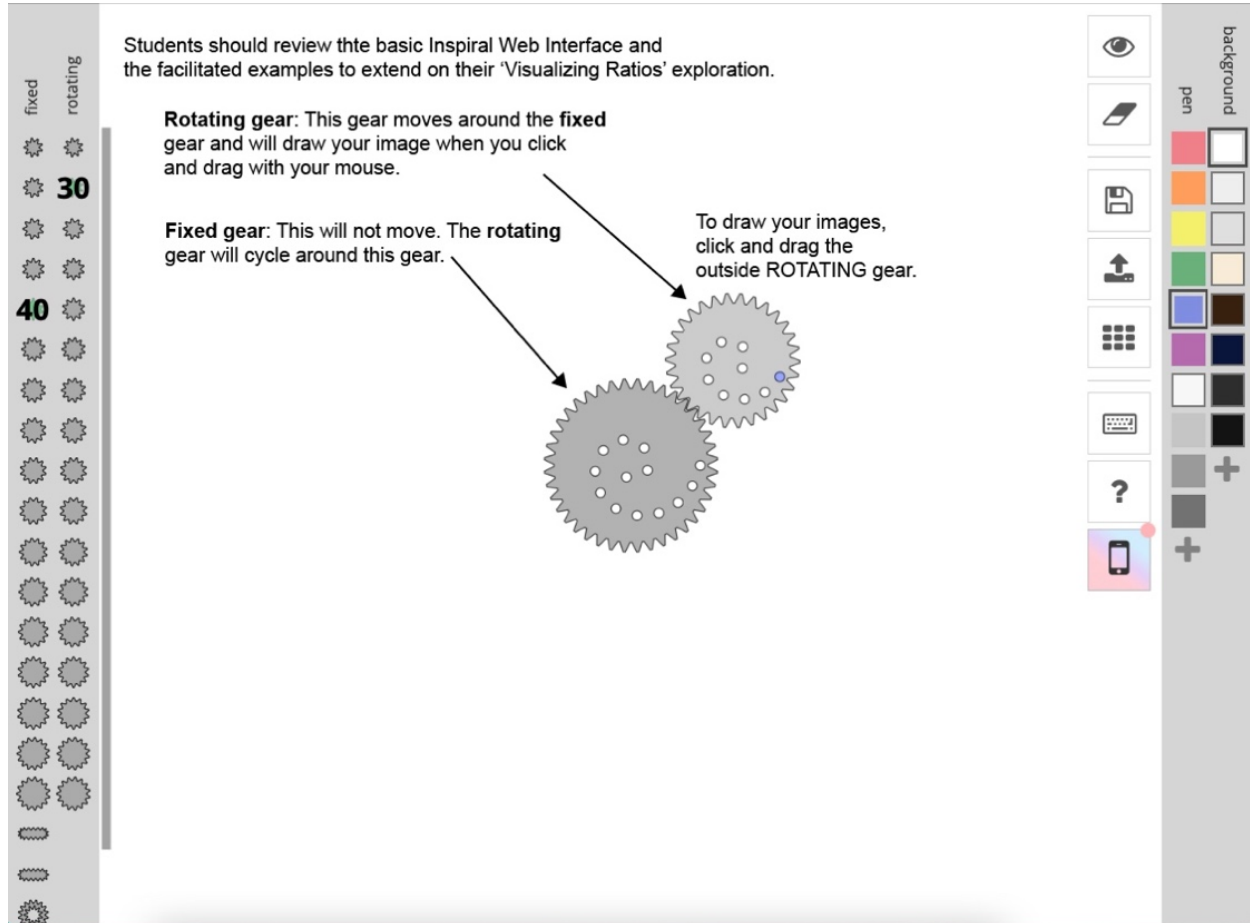   a. *What would it look like if your background was a dark color and your lines were a lighter color?*
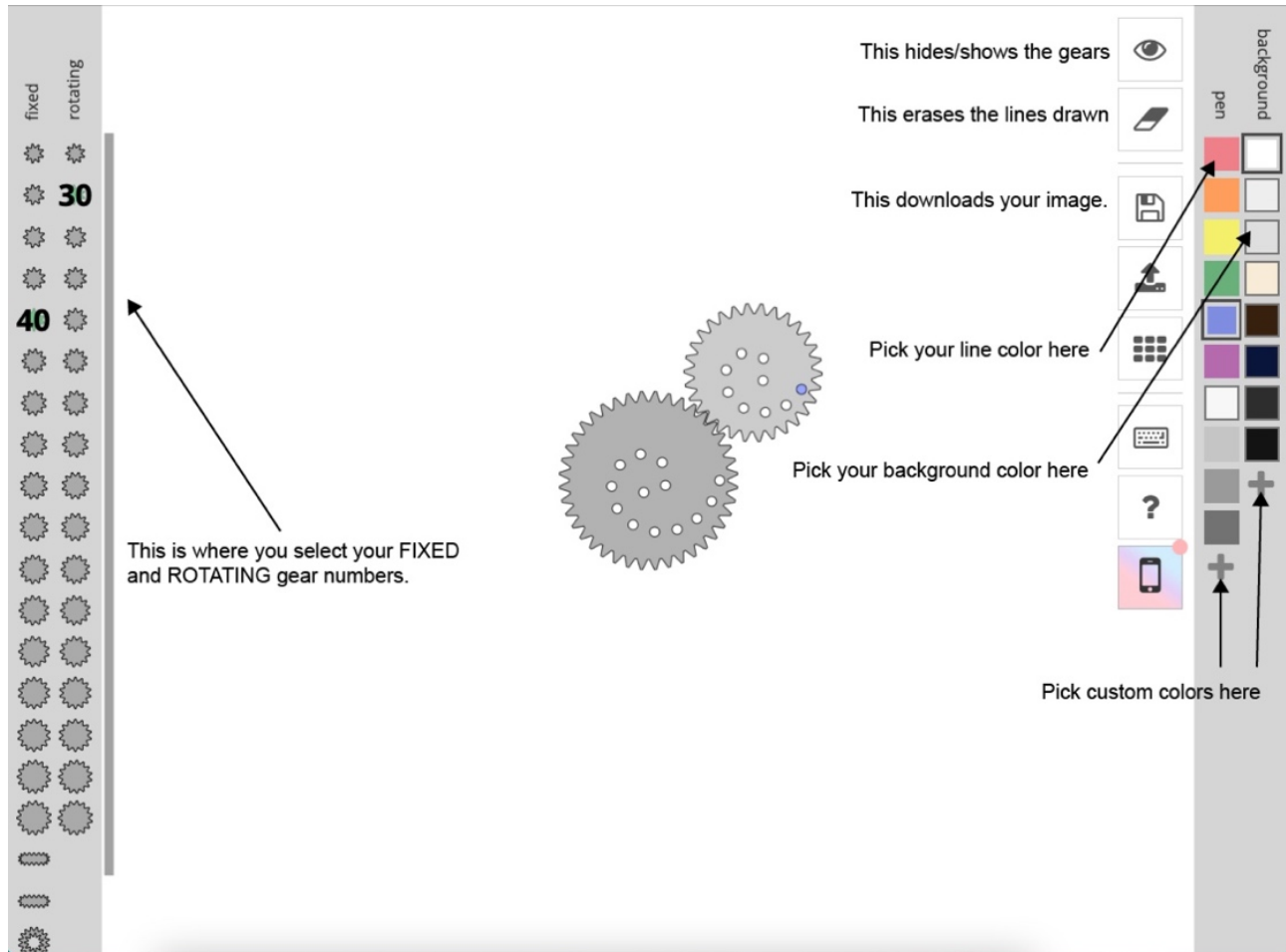
3. To begin making an image, click and drag the outside ***rotating gear***. Be sure to choose the little dot **_closest_** to the edge of the rotating gear.

Students should review thte basic Inspiral Web Interface and the facilitated examples to extend on their 'Visualizing Ratios' exploration.

Rotating gear: This gear moves around the **fixed** gear and will draw your image when you click and drag with your mouse.
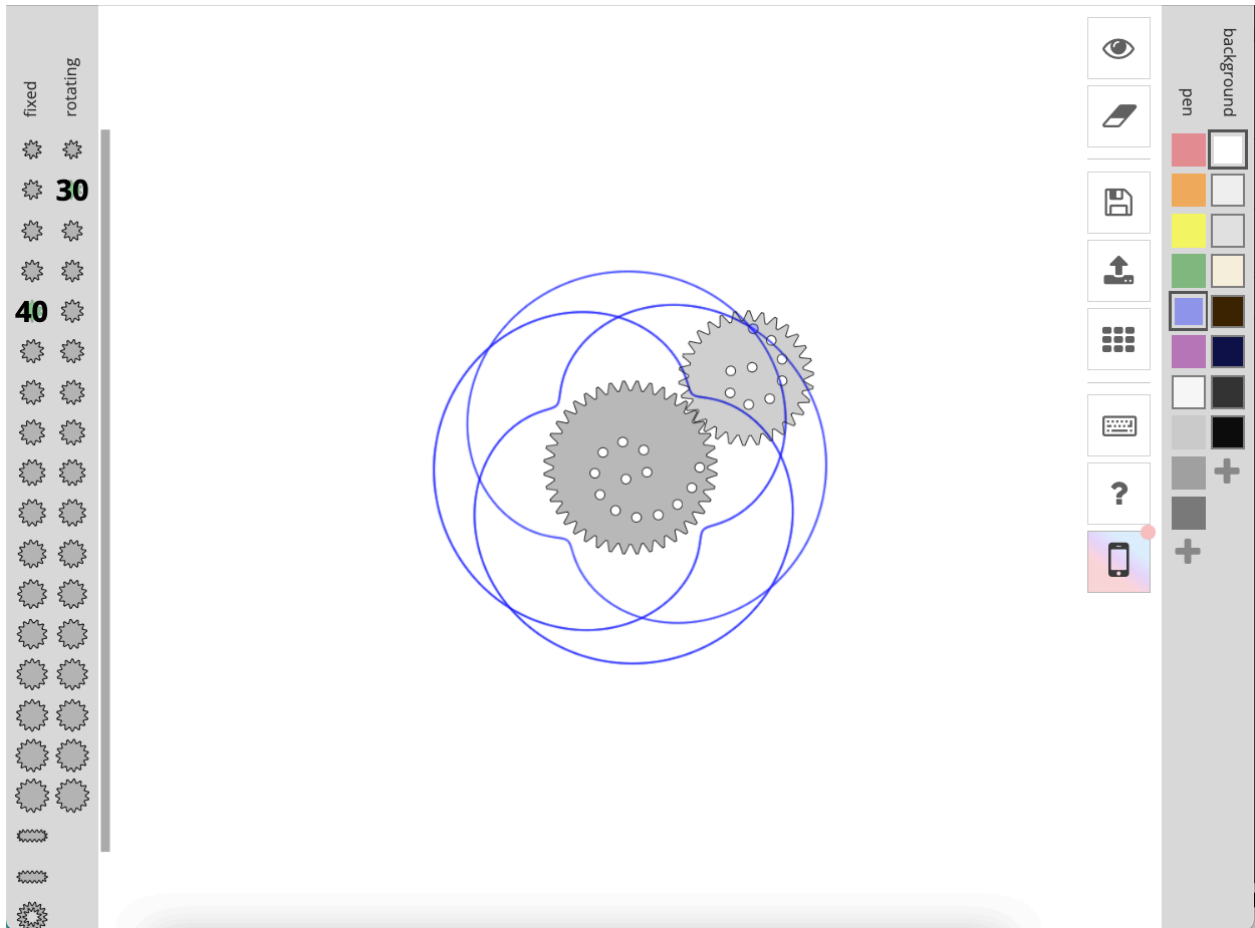
Fixed gear: This will not move. The **rotating** gear will cycle around this gear.

To draw your images, click and drag the outside ROTATING gear.

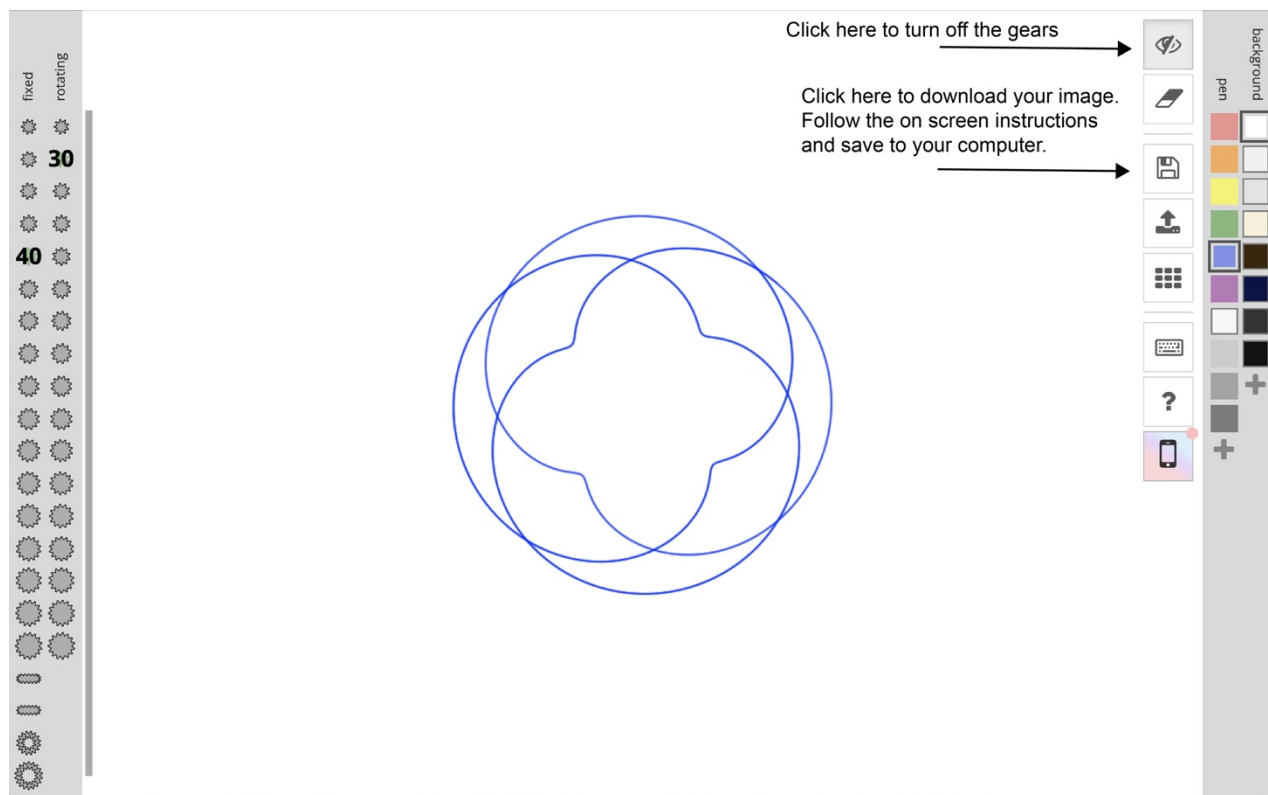fixed   rotating

30

40

background

pen

4. Once you have used your mouse to click and drag, you will see the images start to appear. Keep dragging until the image is complete, or the color stops changing.

5. Once your image is complete, click the eye on the right to turn off the gear set.

Click here to turn off the gears

Click here to download your image.
Follow the on screen instructions
and save to your computer.

6. Click the 'download image' icon and follow the on-screen instructions. When the prompt asks you to name the file, make sure to include the fraction set you used. In this case we will name it '24/30_yourname.'
   a. ***Make sure you save it to a location where you can find your image.***

3. **Print images and discuss variables that affect the shape of the image.**
- Students should print out three copies of each image. One will be used for the worksheet and the others will be used for the final art gallery presentation.

- Facilitate a discussion with the students about how they can use the website to design images with different shapes and colors:
  - What things can you change to design your own images?
  - What happens if you change the gears?

In the next section, students will explore gear sets and how they relate to equivalent fractions

EXPLAIN #1: Visualizing Equivalent Fractions Using Gear Sets

Section Goal: Students will use their new knowledge of gears, teeth and the spirograph website to begin to make the connection between gear ratios, equivalent fractions, and the spirograph image.

Section Activities:

3. **Introduce, or review, the concept of Equivalent Fractions.**

   ○ Students should understand 4.NR.4 section 2:
     ▪ Compare two fractions with the same numerator or the same denominator by reasoning about their size and recognize that comparisons are valid only when the two fractions refer to the same whole.

   ● **Direct students to fill out the <u>Visualizing Equivalent Fractions Worksheet</u> for their images.**

     ○ Students should tape one of their images in the box on the worksheet and record the number of teeth on their two gears.

     ○ Students should then mathematically reduce the fraction.

   ● **Engage in class discussion**

     ○ What links can you make between equivalent fractions and the imagery produced?

In the next section, students will explore how to use equivalent fractions with the Inspiral Web Interface to generate new images.

EXPLORE #2 – How equivalent fractions make similar imagery

Section Goal: Students will use the web interface to learn about how to visualize equivalent fractions.

Section Activity:

1. **Students will explore images created using equivalent fraction gear ratios.**

   ● Using the Inspiral Web Interface, have all students generate an image using a gear ratio of 24/30, using a ROTATING gear of **24** and a FIXED gear of **30**.
   ● On their Visualizing Fractions Worksheet, students should paste in their image and solve for the reduced fraction.
   ● Discuss what other gear pairs would have the same gear ratio.
     ○ Example: **32/48, 40/60, 48/72** all reduce to **2/3**.
   ● Have the students generate additional images with the same 2/3 reduced gear ratio, using different size gears.

- Once finished, have the student print and label the new images.
- Ask the students what similarities and differences there are in these new images.

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EXPLAIN #2

Section Goal: Students will make the connections between the gear set ratios and how they affect the pattern of the drawings they make, in a predictable way.

Section Activity:

- **Discuss with students the connections between the equivalent fractions sets and the visuals they produce.**
  What similarities do they see?
  What differences do they see?
  Note: When gear sets have the same ratio but a higher number of teeth per gear, the images pattern stays the same, but the size of the image changes.

  In the next section, students will return to the challenge and create a new piece of art

ELABORATE

Section Goal: Students will expand on the concept of visualizing fractions and constructing their own exploratory images. They will elaborate on their knowledge and skills by investigating new combination of **FIXED** and **ROTATING** gear sets.

Section Activity:

- **Return to the original challenge:**
  - *Your principal would like to exhibit some new art in the hallways and has asked your class to provide some math-based art that utilizes fractions and a computer program. So, you will be creating digital, math-based art.*

- **Facilitate student understanding with the following questions:**
  - Questions
    - What are some different methods that can be used with the Inspiral Web Interface to modify the images?
    - What are potential ways to make the images more elaborate?
    - How do the equivalent fractions set affect the size of the images?

- **Students should return to the challenge and create new artwork, experimenting with different number sets, colors, etc**
  - Note: Encourage the students to try to make images that are NOT based in equivalent fractions, to show them how the number sets that affect the resulting image.

In the next section, be sure to measure student understanding and completion of the activity. Additionally, prepare the students for making their images presentable for their hallway display.

EVALUATE Wrapping Up

Section Goal: The final product of the lesson will be a presentation in the form of an artistic gallery show. Students should engage in meaningful discussion and reflect on their learning.

Section Activity:

- **Students will use copies of their printed images and mount them to a black card stock or foam board and hang them in the hallway outside their classroom.**

- **Potential discussion topics**
  - Identify images that used the same equivalent fraction gear ratios.

  - What different techniques did people use to generate different looking art?

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